Does Enterprise Value Really Depend on WACC and Free Cash Flow?
The Evidence of Irrationality from the Oil and Gas Sector

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Abstract

The main objective is to check whether traditional DCF model, based on stable rational expectations for cash flows and discount rates really works in an intermediate term—from a quarter to three years. The sample was formed from six major companies of oil and gas sector. The main conclusions are—changes of enterprise value are independent of the changes WACC, free cash flow, and operating cash flows. This may be explained by the impossibility to make durable assessment neither for expected cash flow nor for the discount rate, which in fact means failure of strongly rational models like CAPM or MM. To handle out this implied irrationality the new model proposed, based on the stochastic cost of capital, which follows the model of generalized method of moments by J. Cochrane.

Keywords: enterprise value; WACC; stochastic discount rates; generalized method of moments; rationality; behavioral economics

JEL classification: C65, G32

How cash flows and cost of capital should influence enterprise value if MM theory was realistic?

The original motivation for this work is the question—how can traditional DCF approach (based on MM theory) explain real empirical middle-term changes in enterprise value? And particularly interesting is intermediate term—from a quarter to three years, which conforms to typical investment horizon for financial investments. Similar questions are discussed in the numerous investigations, but mainly for the long-term horizon and in the context of optimal capital structure, like in Bhamra, Lars-Alexander, and Strebulaev (2010) (for wider reference list see Koller, Goedhart, and Wessels (2010), or handbook by Pratt and Grabowski (2008)).

There is well known (e.g. Cochrane, 2005), that at very short-term periods (like days or weeks) price movement conforms to a random walk (or martingale). Also, there are empirically justified long-term returns on stocks and indexes (e.g. Hansen and Heaton, 2008; Chen and Hill, 2013) which are different for different periods because of business cycles and macroeconomic shocks. That may be explained by a very evident reason—growing companies should grow both in cash flows and value. And falling companies should fall in both dimensions too. However, that is not so evident for free cash flow, as growing companies (e.g., Apple) may reinvest in the growth major part of its operating cash flow and their free cash flow may not change.

And for an intermediate horizon (here it is implied from one quarter to three years) the relations between cash flows and value still is not researched well and for a good reason, because this is a puzzle. Moreover, for medium term (up to 5 years) that is a puzzle too. However, the intermediate term from one quarter to three years may conform to typical investment horizon for majority of financial investors and it may be supported by maximal samples of data, available in "Bloomberg" (e.g., for such externalities, as WACC, FCF, CFO, M-cap and etc. for the period 2000–2017).

MM theory and interrelated CAPM are widely and rightly considered as the basement of modern financial theory (see, for example, Pagano, 2005) and both have some different modern variations. The third "cornerstone" probably is Black-Sholes theory. All the three are widely applied and all three are widely questioned.

There are several possible ways to prove MM theorems. Most of them usually follow original approach as it was proposed by Modigliani and Miller (1958), and later, in a corrected form by Modigliani and Miller (1963). Commonly proofs of MM theorems are based on the impossibility of arbitration and thus assume financial markets as perfect, all-knowing and being always right. Perhaps, the most simple and elegant proof is proposed in the academic textbook (monograph in fact) by Tirole (2006) in a context with an extensive literature review on the MM theory, its latest versions, redactions and empirical verifications.

Stiglitz (1969) criticized MM theory stating five limitations of its proof, and particularly (at number five) that it was not clear how the possibility of bankruptcy affected the validity of MM theorems. Actually, MM authors and their followers did not consider the costs of bankruptcy (or financial distress) as a significant factor. So, the main direction for following MM research has become tax effects accounting—see Miller (1988). But still, many authors rightly considered the impossibility of bankruptcy (going-on concern) as a controversial. The contradiction is evident—if capital structure consists of debt only, then equity is zero and firm should go bankrupt both theoretically and in practice. Then a cost of bankruptcy was later embedded in the MM theory by "trade-off" theory as an exogenous factor.

Merton (1974) provided a specific model for the cost of bankruptcy, based on Black-Sholes theory, which still is in use by Moody's KMV model. However, Merton's work is widely (and rightly) recognized as classical and brilliant, it also consists of some contradictions and controversial approaches. For example, Merton claimed that MM theory works even under bankruptcy which is evidently wrong (see above).

The practical cornerstone of Moody's KMV model is the use of "implied" volatility instead of real volatility. And that "implied" volatility (Moody's KMV model use mainly "EDF" and "distance to default") actually is calculated back with Black-Sholes model by an empirical database which is confidential by commercial reasons. So, employing that database is the valuable secret of Moody's KMV model. The same method is applied in the Black-Sholes model for options. In the work of Zhukov (2014), there were shown, that if use real volatility instead of "implied" one there is no relation between volatility and return.

Many authors are referring to the classical monograph by Donaldson (1969), where discussed various financial strategies used by US companies and many practical examples. Donaldson concludes that companies usually stick to permanent capital structure and if change it (which is done under circumstances only), then follows the certain hierarchy of decisions. Later, his theory gets the name "pecking order theory". Following Donaldson results, Myers (1984) introduced a new direction in theory development—accounting for transaction costs. However, Myers denied the materiality of bankruptcy costs as authors of MM theory did.

Empirical check for both trade-off and pecking order theories in the view of capital structure was run by Fama and French (2002). As a result, the panel was obtained with conflicting conclusions, where some (3 conclusions) are rather in consent with the first theory, and some another (3 conclusions) are rather in consent with the second one. So the final judgment was made that both theories may be right (despite logical contradictions in the basic assumptions).

An alternative method of stochastic modeling was proposed by Strebulaev (2007). It was based on modeling a random change of enterprise value and then changing capital structure for better. That method of stochastic modeling largely is based on Merton's model for valuing bonds and default costs as an option. Strebulaev finds the contradiction between the theories of "compromise" (trade-off) and "hierarchy" (pecking order) is inconsequential in terms of modeling results. Thus, the conclusion was evolved that results of Fama and French (2001) are not controversial as it seems and really both theories may be right.

Evidently, the limited lifetime of the company may influence its WACC and the enterprise value compared with a firm unlimited in time, as it supposed to be under the "going-concern" concept. That was an initial idea for the theory of Brusov-Filatova-Orechova, proposed in Brusov et al. (2011). It was based on optimizing the enterprise value

with a limited lifetime, versus a model firm in the MM theory which was denoted as "perpetuity" firm.

An alternative view of the corporate financial policy was exposed by Tirole (2006), where the central point is an agency conflict and related information asymmetry between insiders (managers) and outsiders (shareholders). That is certainly one of the main sources of market imperfectness and with no doubt one of the central (if not the main) problem for the corporate finance. The agency conflict obviously may have an impact on both costs of bankruptcy (or financial distress) and transaction costs. The monograph is based on the wide review of empirical results and considers some contradictions between empirical results and financial theory. That goes well beyond of either MM or CAPM theories, even modernized.

The common identity for the enterprise value (e.g., see Damodaran (2008)) is:

$$EV = MV(Eq) + MV(ND) \qquad (1)$$

The second term represents the market value of the net debt and usually, it is the difference between gross debt and liquid assets — cash and market securities.

$$MV(ND) = MV(D) - Cash - MS \qquad (2)$$

From the postulate of investor's rationality there can be deduced that enterprise value is equal to the discounted value of the free cash flows to the company:

$$EV(t0) = \sum_{t=t0}^{\infty} FCF(t)/(1 + CC(t))^{t-t0} \qquad (3)$$

Here FCF(t) — the expected future free cash flows to the firm, and CC(t) is the expected cost of capital. EV(t0) is the enterprise value at the moment t0.

The company's cash flow (free cash flow) is usually defined as the cash available for distribution to investors and creditors after capital expenditures:

$$FCF(t) = CFO(t) - CAPEX(t) + Int(t)(1-T(t))$$

Here CFO(t) is net operating cash flow; CAPEX(t) — net investment in fixed capital; Int(t) — interest for the loan; T(t) — the effective tax rate applicable.

Identity (3) is also widely used as justification for the company's valuation methods on discounted free cash flows or DCF (e.g., see monograph by Koller, Goedhart, and Wessels, 2010). Accordingly, both in theory and in practice, the central problem of the company value management is usually reduced to cash flow management, risk management or capital structure management.

Formally (3) may also be treated as identity, or as an equation for either unknown cash flow or capital cost. However, as there is only one equation, one may find either single average cash flow for the given cost of capital or single average capital cost for given average cash flow or given both find enterprise value and etc.

MM theory in fact (albeit implicitly) use the postulate of a rational behavior of investor which create enterprise value (3) by the market equilibrium price. And it proves that the discount rate in (3) is equal to the weighted average cost of capital WACC, composed from required return (opportunity costs) on shareholders' equity and the required yield on debt, taken after tax shields:

$$WACC(t) = Re(t) \times we + Rd(t) \times wd(1 - T) \qquad (4)$$

Here Re — required return for equity (commonly treated as a return to a diversified portfolio with the same risk and leverage); Rd — the cost of interest-bearing long-term debt; *we* and *wd* — shares of equity and debt in the enterprise value; T — marginal rate of corporate income tax.

As a rule of thumb, the weighted average cost of capital (4) is considered permanent in the MM theory since all variable factors (including individual risks) are counted in the expected free cash flows.

Expression (4) for the discount rate, and especially combined with CAPM for the cost of equity may be considered as the very arguable part of MM theory. But in fact, the expression (3) is arguable too.

However (3) may be considered just as an identity for enterprise value, as it directly follows from the postulate of the rational behavior. Also, identity (3) can be used as the equation for average discount rate given the cash flows or as the equation for average cash flow at a specified discount rate and growth rate. In the work by Zhukov (2015), there was presented another alternative and generalized variant for MM theory, including all bankruptcy

(default, financial distress) costs, and transaction costs as adds to discount rates, covering both trade-off and pecking order theories. There is shown that modified MM theory can be built just on identities (3) and (4), added with an assumption that required yield on equity depends linearly on the debt leverage. The latter follows directly from the effect of financial leverage. So, in fact, MM theory actually relies neither on the impossibility of arbitrage, nor on "going concern", as it is widely accepted, but just on (3) and (4) identities, which may be derived almost directly from the postulate of rational behavior for investors.

The important generality of (3) is—free cash flow is a stochastic process and may depend on the time (t). For the median value there may be considered any of a reasonable trend (linear, exponential, declining and etc.). And the stochastic error may have any distribution but must have zero median. But the central question for the (3) is—may either cash flows or discount rates depend on the reference point (t0)?

This is important because investors' expectations regarding cash flows and risks may change. Formally in the (3) and (4), this independence is not required, as the model is designed just for an assessment at the current moment (t0) and every time vision of future may change. But there are two central points—how that change may be reflected in a model and how it may affect outcomes?[1] Formally if μ is probability measure and $\lambda(t, t0, \mu)$ is probability density distribution for FCF, then:

$$\text{FCF(t, to)} = \int_{-\infty}^{\infty} \lambda(t, t0, \mu) d\mu$$

However, if the mathematical expectation of stochastic cash flows FCF(t, t0) may change with the reference point of time (*t0*) then this parametric set of stochastic processes (*t* is a parameter) is not stationary with time (t0). And then the cash flow as the stochastic process cannot be represented as a sum of a trend functions of future moment *t* and any stochastic process (may depend on *t0*) with zero median. Which means—FCF(t, t0) may

form stochastic process with unknown and changing variance, unknown and changing median and also with unknown and changing trend.

All that means that investor's appraisal at any time *t0* is not durable (prudent) and as unreliable as the bet in the casino. But if transaction costs in casino come lower than in the stock market (due to lower costs of expert services), the stock market would go bankrupt. So far, as the stock market still survive the competition with a casino, it means that either it provides lower transaction costs or that it provides some other service besides casino do. For that reason, if rationality of investors prevails over irrational (changing) assessments, expected free cash flows and discount rates in (3) must be independent on reference point *t0*—the moment of appraisal. And this important statement forms the concept denoted here as "strong rationality".

Definition 1. Strong rational model (for a strong rational investor) for the enterprise value is any model, based on identity (3), where mathematical expectations for free cash flows to the firm (stochastic) and capital cost (fixed) both may depend on the time, but do not change with relevance point t0.

So strong rationality assume that investor does not change its assessment (forecasting) of cash flows and discount rates through the time. The strong rational model means that investor may reliably assess future expected cash flows and then this assessment should not change with time.

The postulate of the strong rational behavior was widely criticized by behaviorists (see, e.g., Richard Thaler, 2016), but as Tirole (2006) sentenced — there is no rigorous model created to include irrationalities (except maybe Beta-Delta discounting). By the other hand, as R. Thaler stated (in the cited above book), many academicians of traditional school believes, that gesture by "invisible hand" of the market may make those rational models work. Also they evidently believe, that market price is "always right" and "fair", and market itself is "perfect", "all-knowing", and finally benevolent, while unpredictable and incomprehensible[2]. So far, applicability of the strong rational model (3), (4) for the empirical data may be the good test for rational behavior postulate.

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[1] The author believes that the same questions were probably stumbling-block for the Merton (1974) work on the bond price. But probably that ideas was initially for the Merton's (1972) model ICAPM which is widely underestimated because experts in finances use multifactor models (see Maio, 2012) while academicians prefer "rational" CCAPM.

[2] These points may pose a good reason to build a cathedral for Holly Market in Chicago University where anybody may pray to the Saint Greed for Benevolent Fair Price. However one must differentiate science and religion.

On the other hand, it may provide a good starting point for the construction of alternative models if necessary.

One of the possible alternatives is to use stochastic discount rate instead of stochastic cash flows in (3). Cochrane (2011) proved that the main role in the prices volatility plays the volatility of discount rates but not the volatility of cash flows. However, those results are very general as the study was conducted for stock exchange indexes, and over a long period of time. So their applicability to individual companies and especially in a medium term is not clear.

Two methods employed—fixed discount rates with stochastic cash flows and fixed cash flows with stochastic discount rates

At the first step the strongly rational model (3) and (4), related to the MM theory (either common or generalized) was examined for applicability to empirical data. Specifically, panel research applied to find the dependence on medium-term changes of enterprise value and capitalization on medium-term changes of cash flows and WACC. There were examined relative changes which are TS-type (trend stationary) and therefore are subject to the usual F-statistics (e.g., see Hamilton, 1994; Wooldridge, 2002). And the results were certainly negative—"strongly rational" model, which implies future stochastic cash flows with fixed expected value and fixed discount rates fails.

At the second step, there was applied author's modification of the generalized method of moments proposed by Cochrane (2005). The term "generalized method of moments" and its idea comes from similar in form (but different in a purpose) general method for statistical evaluation of the best parameters for econometric models, proposed by Hansen (1982). Cochrane's generalized method of moments originally comes from CAPM "family" of methods, which is heavily based on the long-term rationality of investors. And inside the "family" (which includes CAPM, ICAPM, CCAPM and etc.), Cochrane used mostly CCAPM, which is the most "rational" as it is based on the Arrow-Debreu model for global economic equilibrium which implies very restrictive assumptions. Tirole (2006) asserts that the entire MM theory may be obtained from the same Arrow-Debreu model as

well[3]. So all the modern financial theory may be evolved from the global economic equilibrium.

Cochrane (2005) used CCAPM for theoretical excerpts, while specified that theoretical approach leads to unrealistic estimates of internal parameters and CCAMP is poorly applicable to practice. And there are stated some contradictions of CCAPM outcomes with practice, which Cochrane call "puzzles", while actually, the greater puzzle would be if there were found evidence for practicability of CCAPM[4].

And (citing Cochrane) if CCAPM was really applicable to the practical economy, then economic theory may be considered accomplished, particularly because any market price for asset would be assessable with CCAPM (or CAPM, ICAPM, and etc.). Apparently, it is not.

But Cochrane states particularly that there is central equation (5) for that model which does not depend on any highly limiting assumptions (like market equilibrium, stationarity, normal distribution, the impossibility of arbitrage and etc.).

$$p = E(mx) \qquad (5)$$

Here p is expected price and on the right side, there is a mathematical expectation of the product of factor vector (m) and vector of expected future return (x).

So, actually, the model (5) is not affiliated with CCAPM or CAPM or any other theory and it may be taken as the initial model itself. However use of stochastic discount factor "m" in (5) seems quite logical, as it describes a choice between the future and the present consumption.

Generally, in (5) one may choose between two alternative approaches—either to use stochastic cash flows "x" but fixed discount rates "m" (like in MM), or to run with fixed expected cash flows "x" and stochastic discount factors "m". The reason for this duality is evident—any risk factors can be taken into account either in cash flows

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[3] This is not really surprising if apply mathematical logic. By the Gödel incompleteness theorems in a controversial system, any statement may be proved as true, as well as an opposite one.

[4] There is nothing impossible. As Hansen (2017) explained in his latest brilliant book even wrong theory may be justified if use optimization method for internal parameters of the model on a fixed sample. However, does it eliminate the difference between right and wrong theory? Gödel theorems claim it doesn't.

or in discount rates and both approaches should theoretically lead to the same results. Underlying for the first approach is the idea that investor (rational or not) at the time of the evaluation may not be willing to use any future required rates of return which are impossible to predict. Instead, investor rather tends to apply the current required rate of return at the time of assessment. In this case expected cash flows "x" must be considered as stochastic, while discount factors "m" are fixed. If mathematical expectation for the all future cash flows are reliably assessed by investor and expected cash flows and discount rate will not change in future, this model must be denoted as strong rational (by the Definition 1).

All the theories, that assume impossibility of arbitrage actually relies on that case of strong rationality. Particularly, so do MM, CAPM and Black-Sholes theories which constitute the foundation of the modern finances. The same is CCAPM because future consumption becomes unknown (as it really is).

For example, MM theory does not formally require (3) and (4), but it relies on the assumption that risk and expected cash flows for all the companies form the enterprise value and may be assessed for the infinite periods. If this assessment will change with the time than enterprise value becomes unreliable (or irrational) and then so becomes all the MM theory. Formally one may introduce parameter "t0" and assume all that estimates depending on time. But it would mean that market is not in equilibrium and if so, arbitrage is possible and then MM, CAPM (for CAPM case see Fama and French (2006)), and Black-Sholes theories fail. Only ICAPM by Merton (1972) and factor models may still work if they do not require equilibrium.

Some cases of weaker rationality may be captured by behavioral economics. However, Tirole (2006) stated that this new area of economics does not have proper models yet. The well-known behavioral models include hyperbolic discounting, and particularly Beta-Delta discounting model. These models certainly do not conform to the strong rational case even if beta and delta are permanent.

Generally, if either expected cash flows or discount rates in the model may change along with the time of assessment, then the model probably may describe some kind of irrationality. Effectively it means the very simple point—investors are unable to predict neither expected cash flows nor risks for the long future. This point certainly is reasonable and rational from the practical approach, but in traditional theory, it may look like irrationality.

For the second approach, the underlying idea is that investor anticipates average expected cash flows "x" as fixed (determined), possibly with permanent growth rate (positive, zero or negative). For that approach discount rates "m" must be stochastic, which relates to the generalized method of moments by Cochrane (2005) in the form (5). This case may be denoted as "weak rational case" as it implies some of the rationality, but not in the strong form.

Results for the first approach—checking common DCF model (generalized MM theory)

First, represent actual cash flow $FCF^*(t)$, observed at the time (t) as the sum of expected cash flow $FCF(t)$ and stochastic fluctuation $\delta(t)$ with zero median[5]:

$$FCF^*(t) = FCF(t) + \delta(t) \qquad (6)$$

No assumptions about distribution are made here but median must be equal to zero.

If a stochastic series (3) converges in the sense of mathematical expectation, then its sum is equal to the expected enterprise value (e.g. see Wooldridge, 2002).

Under the assumption that free cash flow model (3), (4) is the underlying basis for enterprise value, one get the equation:

$$EV(t) = (FCF(t+1) + EV(t+1))/(1 + WACC(t))$$

This also can be written in incremental form:

$$EV(t+1) - EV(t)(1 + WACC(t)) = -FCF(t+1) \qquad (7)$$

This equation (7) may be extended from the 1 period of time (e.g. quarter or year) to any number of periods.

$$EV(t+1) - EV(t)(1 + WACC(t))^{-n} =$$
$$= \sum_{\tau=1}^{n} FCF(t+\tau)/(1 + WACC(t+\tau))^{\tau} \qquad (8)$$

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[5] No assumptions about distribution is made here but median must be equal to zero.

For the strong rational case (3) and (4), expressions (7) and (8) represent the increase in the enterprise value for the certain period. Further (8) will be considered as the model for increments in enterprise value from 1 quarter to 3 years. Following questions are examined for the selected sample of oil and gas sector:

1. Do the fluctuations in the enterprise value really may be explained with the generalized MM model (3), (4)?

2. Coming away from MM theory—are changes in enterprise value related to the changes in free cash flow, operational cash flow or WACC?

3. If change free cash flow to the operating cash flow plus interest minus tax shields, would it change the answers to questions 1 and 2?

4. To what extent changes in the market capitalization of the company may be explained by the changes in its enterprise value?

For the strong rational case (8) become deterministic. In that case, one may observe that correlation of actual enterprise values and those derived from (8) as equal to 1 and with the significance level (probability of the hypothesis H0) close to zero. However, if that correlation is equal to zero (hypothesis H0), one must assume that investors essentially change their assessment of future risks and (or) cash flows for the every projection period. So, that rational case is not the real one.

The sample for the study was compounded from six companies of oil and gas sector—Lukoil, Rosneft, Gazprom, Novatek, BP, Dutch-Shell. But in order to identify the possible impact of a sample on the results, the company from the opposite sector (by systemic risks) was added—Coca-Cola[6].

To answer the question 1 the validity of the model (8) was examined to the increment in the enterprise value after 1 quarter and up to the 3 years. The result was a conclusion that enterprise value increment for the periods from 1 quarter to 3 years with probability from 35% to 86%, is not correlated with the theoretically expected. So, H0 hypothesis of zero correlation is most likely (or, at least, can't be rejected), and rational case can't explain the real deviation of the price.

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[6] Actually, the sample included more companies from 4 industries, but for the brevity this work describes results for oil and gas sector, adding just one company outside it—Coca-Cola, for comparison.

$$\text{Corr } (EV_{(t+n)}/(1 + WACC(t + n))^n - EV_t,$$

$$\sum_{\tau=1}^{n} FCF(t+\tau)/(1 + WACC(t + \tau))^\tau \sim 0$$

It means also that the common MM model (3), (4) with permanent parameters does not work at the middle-term forecasting period. So, if rational investors use model (3), (4) for forecasting period, they permanently (at least every quarter) change estimates for either the future expected cash flows, or the discount rates, or both together. And these re-evaluations as stochastic process do not make any predictable trend (median), distribution or variation which makes it not stationary with any predictable trend. That is not really surprising because otherwise, the market price was more predictable than it really is. But that does not follow from the efficiency of the market because stochastic changes of cash flows may be considered as a reason for re-estimation of share prices. Eventually, they are not. The re-evaluations of future risk and returns are responsible for deviation of share prices. This is the outcome.

Question 2 is actually a generalization of question 1, but independent on the model (3), (4). Obviously, the cash flows reflect the benefits to investors. So far, under rational behavior postulate, even if investors change their expectations for cash flows and risks, they still should adjust it with any material change in cash flows, or risks, reflected in WACC. And since the cash flows of a company usually can be anticipated from the year ahead-based financial planning, this approach must be based on planned (forecasted) numbers for the future.

Also, there are numerous empirical studies confirming the intuitively obvious assumption that investors adjust prices depending on the news. And news usually relates to either the future cash flows to the company or to the macroeconomic risks. On the other hand, WACC reflects the systematic (common) risks of the company or at least pretend to do it. Accordingly, the change in the weighted average price of capital should lead to a change in the company's value not only for the model (3), (4) but also on the general basis of the rational expectations hypothesis.

But the answer to question 2 is negative—no dependence was observed for the selected sample. It can be assumed that this is true for the many

other companies. A brief description of the results of the study is added in Annex 1.

The relative increment of the enterprise value (in % to the previous) was chosen as the dependent (explained) variable, and as independent (explaining) variables there was chosen relative increments of free cash flow, net cash flows, or WACC. Note that since relative changes were chosen, this process must be TS-type[7] (trend stationary) with zero trend (e.g., Hamilton, 1994; Wooldridge, 2002). Correlation values obtained during regression range from 0.13 to 0.01, but in any case the hypothesis H0 could not be rebutted with a minimally acceptable level of significance (10%), and for the most cases, its probability is higher than 30%.

The answer to the question 3 for oil and gas sector is negative too—results for free cash flow and net cash flow from operating activities nearly coincide. This appears to be related to the relatively stable investment cash flows and is caused by the relative stability of investments in the oil and gas sector which is not the case for fast-growing companies (e.g., for Apple operating cash flow was fast increasing from 2000 to 2017 while free cash flow did not change so much).

However, answer to question 4 is positive—it was found that enterprise value and market capitalization has a strong interdependence for the oil and gas sector, which corresponds to the MM outcomes. Presumably, they are co-integrated, unless a radical change in the capital structure or risks happens. But for the company from beverages sector (Coca-Cola) that result is negative. Probably this difference is caused by the stability of capital structure in oil and gas sector.

These results are surprising and even paradoxical since it is generally assumed (see, e.g., Koller, Goedhart & Wessels, 2010), that investors adjust their estimates of enterprise value either for a change in the cash flows of the company or in the risks. In addition, since the expression for WACC (4) presumably reflects systematic risk, it turns out that the main role for the variability of enterprise value plays idiosyncratic (individual) risks, to the contrary of CAPM or CCAPM theory (e.g., see Sharpe, Alexander, & Bailey, 1999).

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[7] Trend stationarity assumes zero median and fixed variation of error but does not require normal distribution, so it is often considered as a weak case of stationarity.

Specifically, consider the result that change of enterprise value is independent on those of cash flows or WACC. That looks like a puzzle itself because cash flows should reflect expected a return and WACC should reflect risk, or at least, its major part. The most likely answer may be—medium term fluctuations of cash flows and WACC are stochastic and therefore ignored by the market. If so, then—where the "holly" market get data for its "all-knowing" appraisal? The logical answer may be—investors are not as strongly rational as it assumed in MM model or in the model (3) and (4). Then, they must be using somehow another model to price assets.

Does it mean "irrationality"? Perhaps, but in a very specific way — investors can't forecast future expected cash flows and probably change their expectations or discount rates at least every quarter (year) or maybe even faster. So, if one treats "rationality" as the ability of an investor to forecast future expected cash flows and apply to those fixed (but maybe different for every period) discount rates, then answer is "Yes", it means the absence of rationality in that strong definition (see section 2).

The second step—implied stochastic discount rates

At the second step, there will be considered a model of weak rationality (5), based on stochastic discount factors and fixed cash flows.

For the start, consider the most irrational case of model (3), where all variables change every moment, and then depends on the reference point. Particularly, if (3) depends on t0, then the most irrational model may be represented in a form:

$$EV(t0) = \sum_{t=t0}^{\infty} FCF(t, t0) / (1 + CC(t, t0))^{t-t0}. \quad (9)$$

Here all approximations for the cash flows and for the cost of capital may depend on the reference point, so all of them may change every moment. And, for all t0 greater, than t, discount factor CC(t, t0) may change, which means that model (9) is more general than Beta-Delta discounting model. However, this generality may not be necessary, as there is just one explained variable on the left side.

Therefore the most interesting for practical use case may just use one expected (fixed) cash flow, divided by stochastic rate:

$$EV(t0) = FCF(t0)/r(t0).$$

And stochastic rate $r(t0)$ may be represented as a difference of stochastic implied a cost of capital $CC(to)$ and the permanent growth rate of cash flows:

$$r(t0) = ICC(t0) - g.$$

As it was proved in the previous section, changes of enterprise value are very likely being independent of free (or operational) cash flows and WACC. So it is logical (rational) to use expected free cash flow as determined, but, maybe growing (or declining) along with any of specific trend (linear or exponential or any other). Then stochastic discount rates absorb all the information of price changes. That assumption may be derived from two following hypotheses:

1. Investor ignores random fluctuations in cash flows and instead uses some pre-determined value of the expected free cash flow.

2. The investor uses stochastic (randomly changing) discount rates reflecting either the stochastic risks or changes in the investor's expectations about the growth rate of cash flows in future.

With some basic (minimum) investment, the expected free cash flows of the company must have permanent expected value (as all the risks are considered in the discount rates), but with additional investments, free cash flows may grow. Denote FCFexp expected basic free cash flow, independent of time:

$$FCFexp = E(FCF(t)).$$

In the case of zero growth discount rate R may be found from equation:

$$E(EV(t)) = EVexp = FCFexp/R(t).$$

But with some additional investments average free cash flows (but not necessarily actual cash flows) will grow with changing growth rate $g(t)$:

$$EVexp(t) = FCFexp(t+1)/(R(t) - g(t)). \quad (10)$$

Assumption 1 means that investors may use some pre-determined value for expected free cash flow, but dependent on time. And for every period investor may change required rate of return and

(or) growth rate. Then discount rate is stochastic, although investor at every moment applies a single rate for future cash flows.

Then, for (10) with changing growth rate the stochastic discount rates are:

$$r(t) = R(t) - g(t) = FCF(t + 1)/EV(t). \quad (11)$$

Here $R(t)$ — the stochastic cost of capital, reflecting stochastic risks, and *g* is a changing growth rate for expected free cash flow.

More generally, whatever the assessment methods are actually used by investors, their results can be summarized as the expected cash flow ($FCF(t+1)$) and a stochastic discount rate ($r(t)$). If the cost of capital was equal to WACC, then $R(t)$ from (11) would be equal to WACC as well.

However, the results of the research show that growth rate, derived from WACC, have no relation to the changes in the enterprise value, which makes its use pointless. Moreover, the mean values of WACC are much higher than empirical stochastic discount rates (see table 1) and that difference can't be explained by any growth rate (which is negative for the oil sector). While it is interesting that the standard deviation of stochastic rates still is roughly equal to that of WACC. The same results are shown by other peer companies.

So judging by the empirical data, the optimal option for practical appraisal is to use stochastic discount rates. Explaining its meaning both cost of capital and expected growth rate may be chosen as stochastic variable as well as. Both of them may reflect an assessment of risks and prospects by investors and investors may either assess both of it or just use one stochastic discount rate instead. However, the cost of capital looks like better explainable and logical variable for risk assessment, which may change every period on the basis of new data. And if the cost of capital is chosen as a stochastic variable the change of expected growth rate is an unnecessary complication. Therefore growth rate may be considered as permanent without loss of generality.

Conclusions

1. Model (3), (4) (a generalization of MM theory) is equivalent to the strong rational investor, so it is a strong rational model (see definition 1).

2. Strong rational model (3), (4) does not explain the medium-term changes of enterprise value.

Table 1

Averages and variability of WACC and stochastic discount rates for BP since 2000 by 2016. (according to data from Bloomberg)

	WACC	CFO, mln. $	FCF, mln. $	Rcfo	Rfcf	EV, mln. $	Mcap, mln. $
Median	0.088	539	135	0.012	0.003	144000	111000
St. Var.	0.190	1.840	0.460	0.210	0.210	0.270	0.380

Note. Rcfo — stochastic discount rate (R(t)) derived from CFO; Rfcf — the same rate, applicable to FCF. As free cash flow is four times smaller its discount rate is four times lower.

3. In the medium-term assessment using the WACC calculated in line with the theories of MM and CAPM (or CCAPM), as a discount rate for the free cash flow gives no results comparable with empirical data for a selected sample of the companies.

4. Changes of the WACC do not affect medium-term changes in the enterprise value or the capitalization for the selected sample of companies which means that WACC does not reflect the real cost of capital for the selected sample.

5. As changes of WACC reflect changes in systematic (common) risk, it may be assumed that individual (idiosyncratic) risk (which are not reflected in WACC) provide a major influence on medium-term changes of enterprise value.

6. Medium-term changes in free cash flows do not affect changes to the enterprise value of the companies from the selected sample (which deliberately does not include growing companies).

7. Cochrane's model of generalized moments in the form (5) may be useful to evaluate a company price instead of models with fixed discount rates and stochastic cash flows, like (3), (4).

8. To assess the enterprise value of company the possible way is to use long-term fixed cash flows, growing with permanent growth rate (positive, zero or negative) and stochastic cost of capital.

Appendix

Checking the independence of price changes from the change of the company's cash flow (FCF, CFO) and the discount rate (WACC)

Because the company's value changes constantly in real time, it can be interpreted as a realization of a stochastic process. Changes in the prices of shares in companies usually are of type DS (difference stationary) and to examine them usually there are used autoregressive models (AR), or combined with MA (moving average) process— ARIMA models. In the present work subject of research is the dependence of relative change of enterprise value from the relative change in cash flows and WACC. The percentage change in asset prices refers to processes of type TS (trend stationary) with zero trend and, therefore, results may be assessed with applicable F-statistics. For the percentage change in the enterprise value it is:

$$dEV(t) = (EV(t) - EV(t - 1))/EV(t).$$

Independent variables were changes the discount rate WACC, free cash flow FCF (4) and operating cash flows adjusted for interest payments (10):

$$dWACC(t) = (WACC(t) - WACC(t - 1))/WACC(t)$$
$$dCFO(t) = ((CFO(t) - CFO(t - 1))/CFO(t)$$
$$dFCF(t) = ((FCF(t) - FCF(t - 1))/FCF(t).$$

For example, for the BP Corporation, the chance of hypothesis H0 is over 84%. The only variable which tends to show a sustained and significant correlation with the enterprise value (and with a correlation coefficient close to one) is market capitalization. This conclusion is consistent with MM. However, this conclusion is not trivial, given that in expressions (2) and (3) all variables may change significantly over time. Moreover, for the one company from the sample, Coca-Cola, this conclusion turned out to incorrect—change in enterprise value was not associated with changes in capitalization. The reason for this is not clear, but it is clear that this company proved an exception to the general rule (probably due to the nature of its financial policy).

Also for this company, there are significant dependence of the change in market capitalization and free cash flow changes (for other companies it is not).

Thus, there are no observed significant dependencies of changes in enterprise value from changes to FCF, CFO, and WACC. On the contrary, it is very likely that estimated correlation coefficient is indistinguishable from zero.

Table 2

According full price company (probability of the hypothesis H0 and R 2)

Company	FCF (p-val.)	CFO (p-val.)	WACC (p-val.)	R^2	F-stat (p-val.)	MCAP (p-val.)	R^2 for the Mcap
BP	0,64	0.60	0.78	0.01	0.89	10E-57	0.97
Shell	0.35	0.38	0.50	0.13	0.07	-331.57	0.94
Coca-Cola	0.61	0.65	0.95	0.01	0.97	0.98	1.6E-05
Rosneft	0.63	0.32	0.14	0.07	0,40	2,6E-27	0.94
Lukoil	0.31	0.71	0,40	0.02	0.68	5,1E-45	0.96
Gazprom	0.85	0.24	0.38	0.07	0.23	2,62E-27	0.94

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Разве стоимость компании действительно зависит от средневзвешенной стоимости капитала и свободного денежного потока? Свидетельства иррациональности в нефтегазовом секторе

Павел Е. Жуков[1]

Основная задача статьи — проверка, действительно ли традиционная модель DCF (Discounted Cash Flow — модель дисконтированных денежных потоков), основанная на стабильных рациональных ожиданиях денежных потоков и ставок дисконтирования, работает в промежуточном периоде — от квартала до трех лет. Выборка была сформирована из шести крупных компаний нефтегазового сектора. Основные выводы — изменения стоимости предприятия не зависят от изменения средневзвешенной стоимости капитала, свободного денежного потока и текущего денежного потока. Это может быть объяснено невозможностью осуществления долговременной оценки ни ожидаемых денежных потоков, ни учетной ставки, что фактически означает несостоятельность строго рациональных моделей САРМ (Capital Asset Pricing Model — Модель оценки долгосрочных активов) или ММ (Market Model). В статье предлагается учесть подразумеваемую иррациональность в новой модели, базирующейся на стохастической стоимости капитала, связанной с моделью обобщенного метода моментов J. Cochrane (ОММ; англ. GMM — Generalized Method of Moments).
Ключевые слова: стоимость предприятия; ССП; стохастические ставки дисконтирования; обобщенный метод моментов; рациональность; поведенческая экономика
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